

# Neutron and Proton Structure Functions and Duality

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# Overview

✿ **Quark-hadron duality**: non-trivial phenomenon (see Wally's talk)

Ⓢ **Manifestation**: insight into the dynamic of strong interactions  
*Standard tests of quark-hadron duality*

Ⓢ **Application**: could be used to access kinematic regions otherwise inaccessible

*Use averaged resonance region data to constrain PDFs at large  $x$ ?*

✿ **Experimental tests of quark-hadron duality** in:

Ⓢ proton  $F_2^p$  structure function

Ⓢ neutron  $F_2^n$  structure function

↳ New method: extract  $F_2^n$  from nuclear  $F_2$

- Application of method to smooth curves

Y. Kahn, W. Melnitchouk, S.A. Kulagin, Phys. Rev. C 79, 035205 (2009)

- Application of method to data + Quark-Hadron Duality in  $F_2^n$

S.P. Malace, Y. Kahn, W. Melnitchouk, C. Keppel, Phys. Rev. Lett. 104 102001 (2010)

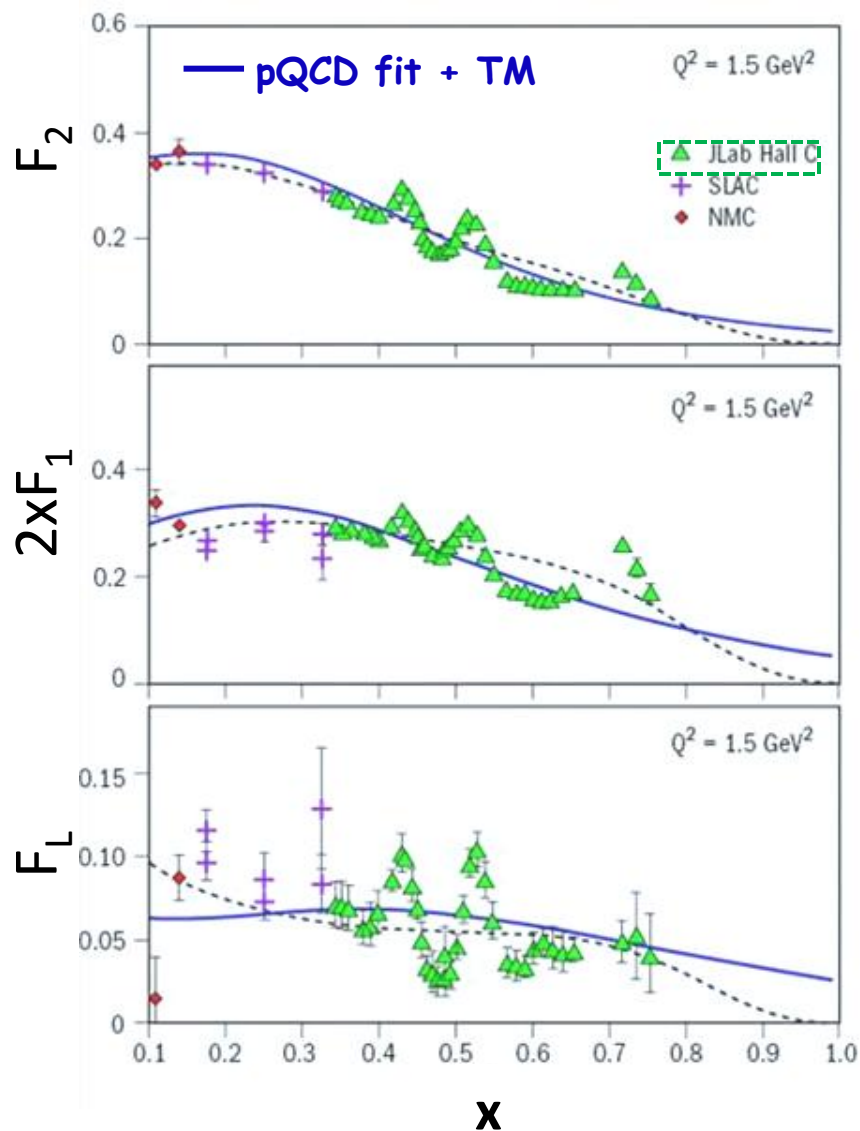
- Application of method to data: (a lot of) technical details

S.P. Malace, Y. Kahn, W. Melnitchouk, in preparation

✿ **Plans for future**

# Quark-Hadron Duality

Y. Liang *et al.*, nucl-ex/0410027 (2004)



✿ On average, the resonance region data mimic the twist-2 pQCD calculation (pQCD + Target Mass corrections)

→ This happens at a **surprisingly low  $Q^2$**



*“The successful application of duality to extract known quantities suggests that it should also be possible to use it to extract quantities that are otherwise kinematically inaccessible.”*

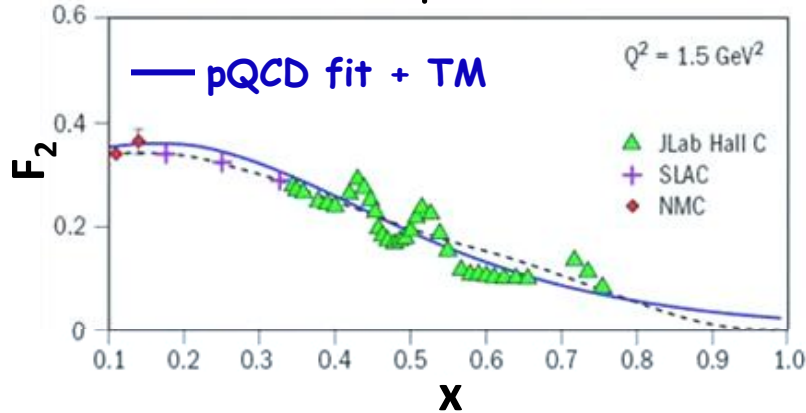
*(CERN Courier, 2004)*



✿ **Quark-Hadron Duality:** needs to be verified and **quantified**

# Standard Tests of Quark-Hadron Duality

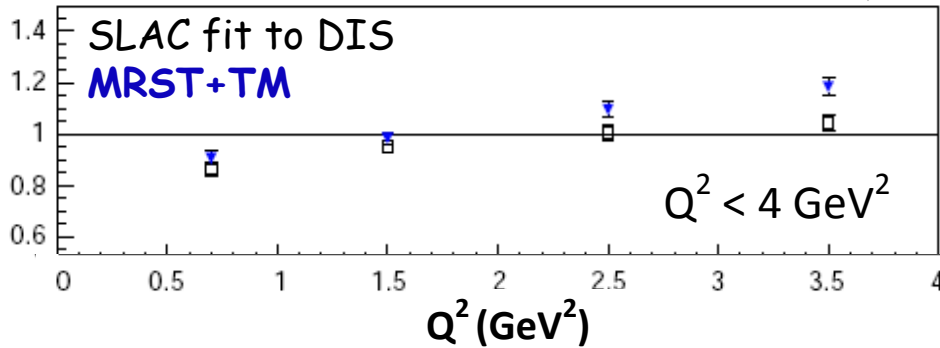
Basic test of Duality: the  $Q^2$  behavior of averaged resonance region data when compared to QCD calculations



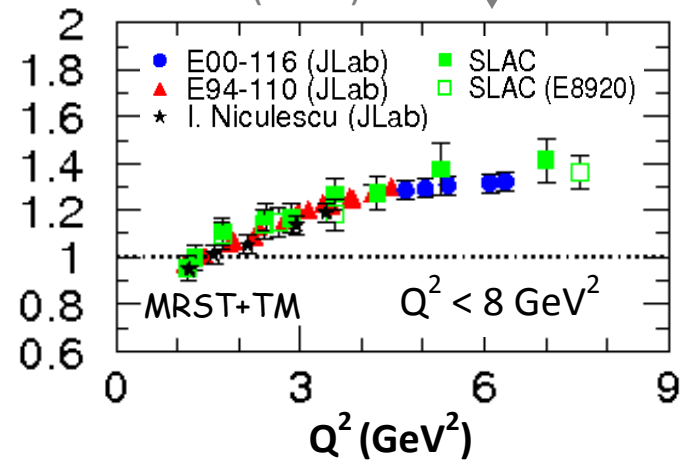
$$\frac{\int_{x_m}^{x_M} F_2^{data.}(x, Q^2) dx}{\int_{x_m}^{x_M} F_2^{calc.}(x, Q^2) dx}$$

Example: integrals over RES Region ( $W^2 < 4 \text{ GeV}^2$ ); comparison of data to MRST+TM

Y. Liang *et al.*, nucl-ex/0410027 (2004)



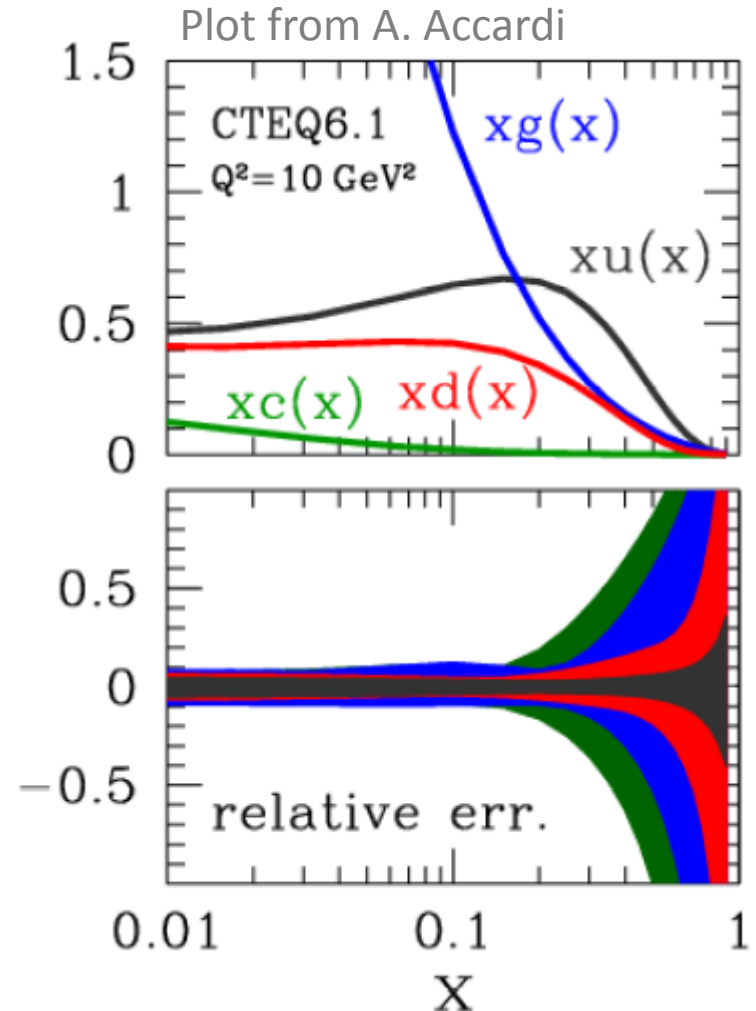
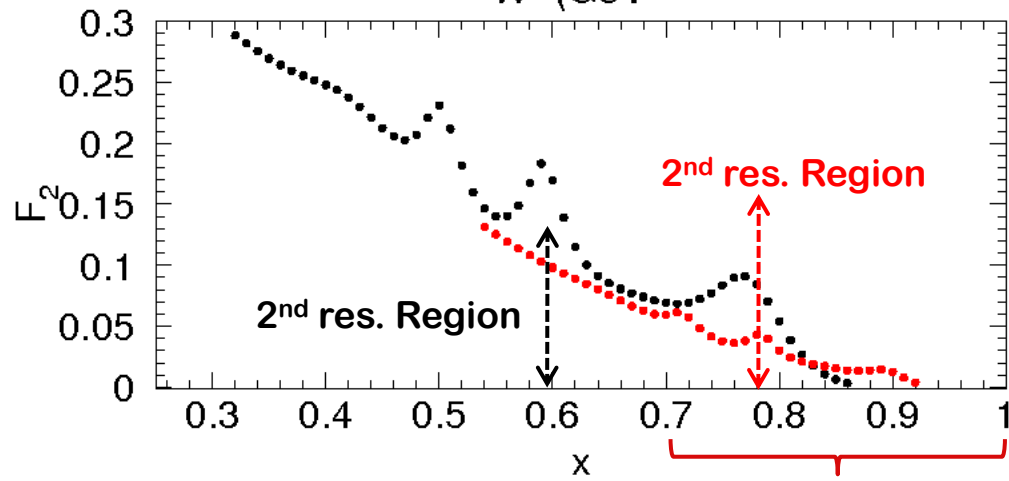
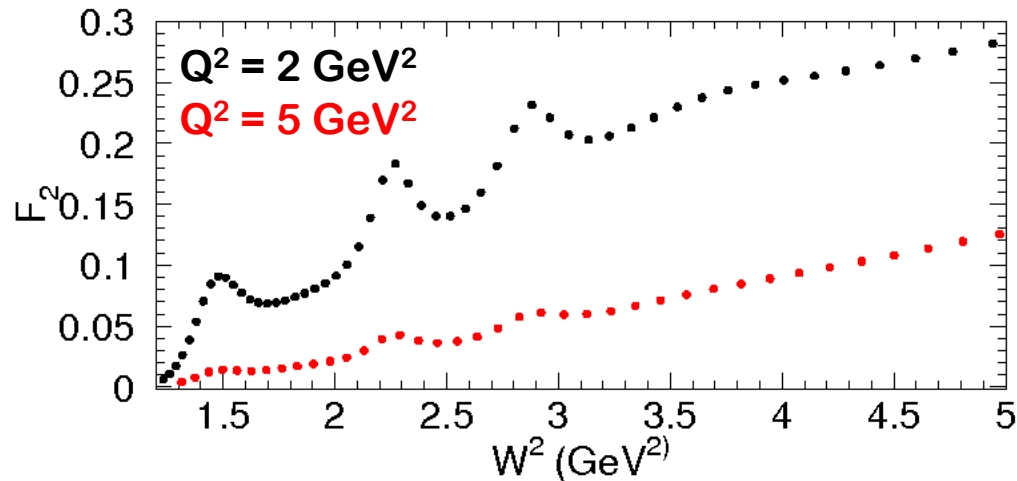
S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)



- 2004: agreement better than 5% at  $Q^2 = 0.5 \text{ GeV}^2$  but  $\sim 18\%$  at  $Q^2 = 3.5 \text{ GeV}^2$
- 2009: deviation of data from MRST+TM increases with  $Q^2$  up to  $Q^2 \sim 4.5 \text{ GeV}^2$  then saturates

# Tests of Quark-Hadron Duality at large $x$

- Kinematics: with increasing  $Q^2$  resonances slide in regions of larger and larger  $x$



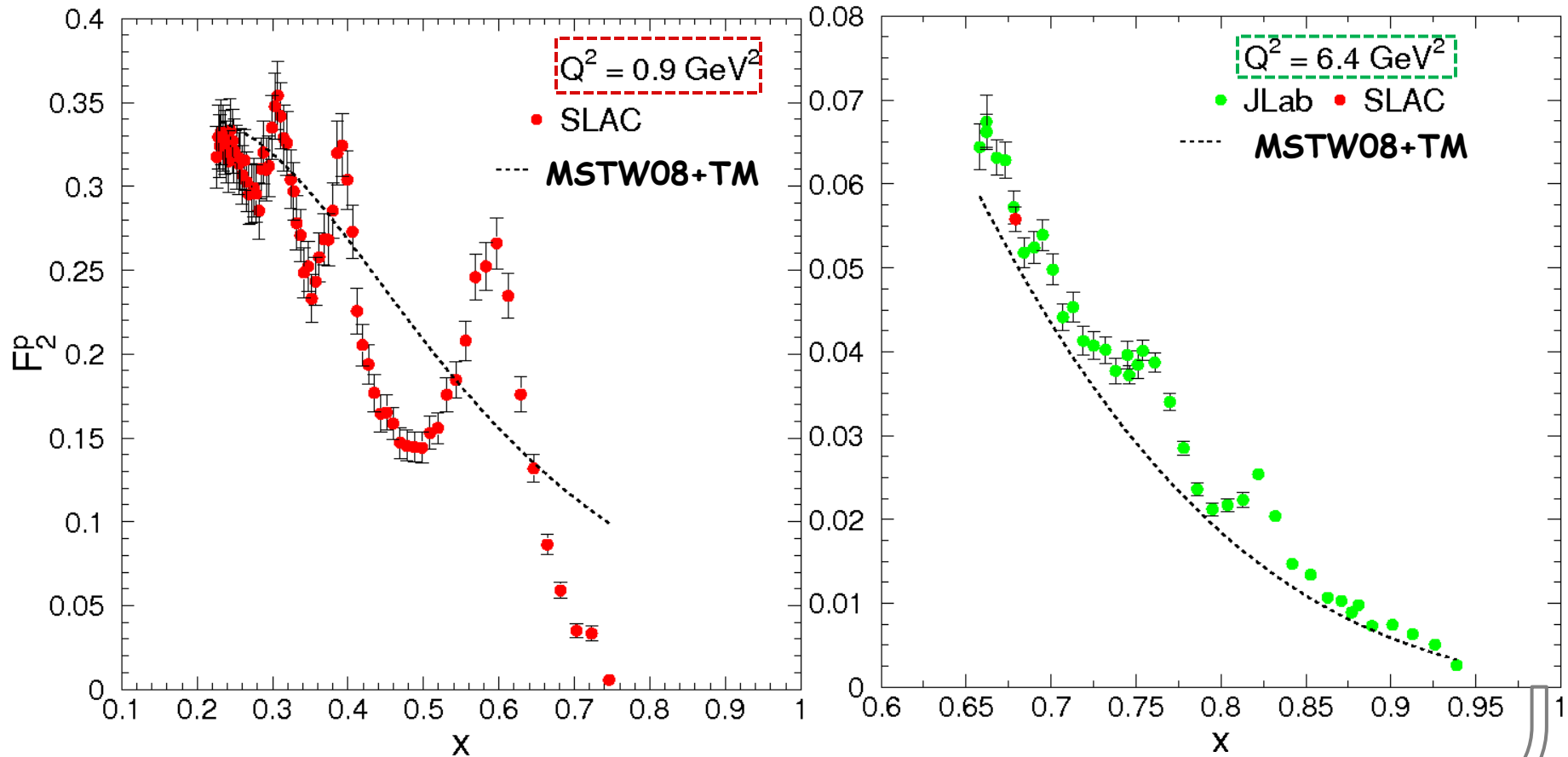
- PDFs (CTEQ, MRST, MSTW) poorly constrained at large  $x$

# Tests of Quark-Hadron Duality at large $x$

✿ It is not surprising then:

---> **though** RES data DO average to MSTW08+TM at  $Q^2 = 0.9 \text{ GeV}^2$ ,  $x \sim (0.25, 0.7)$

---> RES data DO NOT average to MSTW08+TM at  $Q^2 = 6.4 \text{ GeV}^2$ ,  $x \sim (0.7, 0.95)$



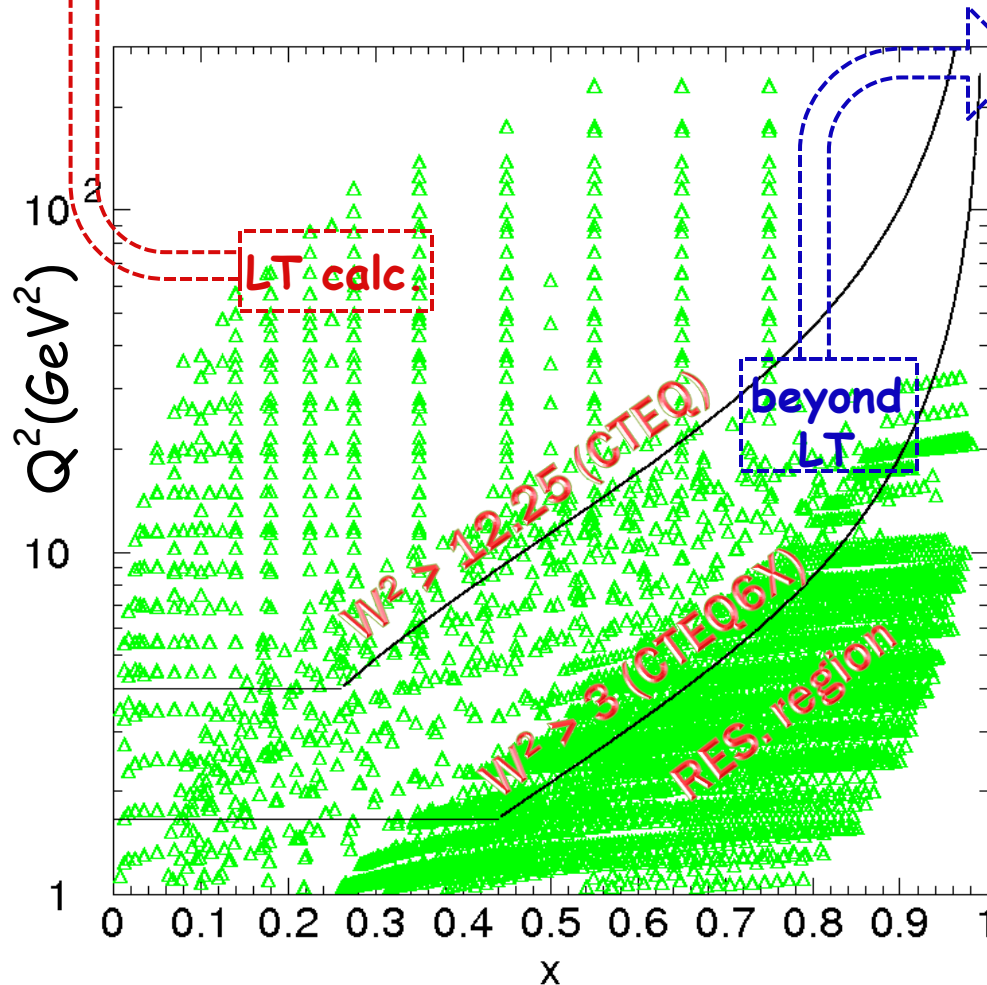
**Not** a violation of duality but very likely due to an underestimation of large- $x$  strength in the pQCD parametrization

# Tests of Quark-Hadron Duality at large $x$

What should we use for quantitative tests of Duality at large  $x$ ?

✿ Leading Twist (LT) calculations  $\Leftrightarrow$  PDFs constrained up to

✗  $\sim 0.65-0.7$ : CTEQ, MRST (MSTW)... 



✿ Calculations beyond LT  $\Leftrightarrow$  PDFs constrained up to  $x \sim 0.8-0.9$

*Alekhin et al.*

S. Alekhin, Phys. Rev. D 63, 094022 (2001)

...

S. Alekhin, J. Blumlein, S. Klein, S. Moch, Phys. Rev. D 81, 014032 (2010)

**CTEQ6X**

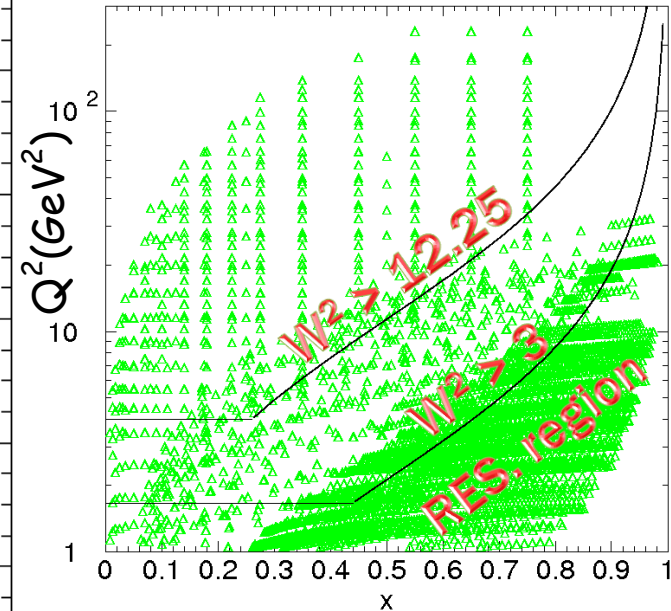
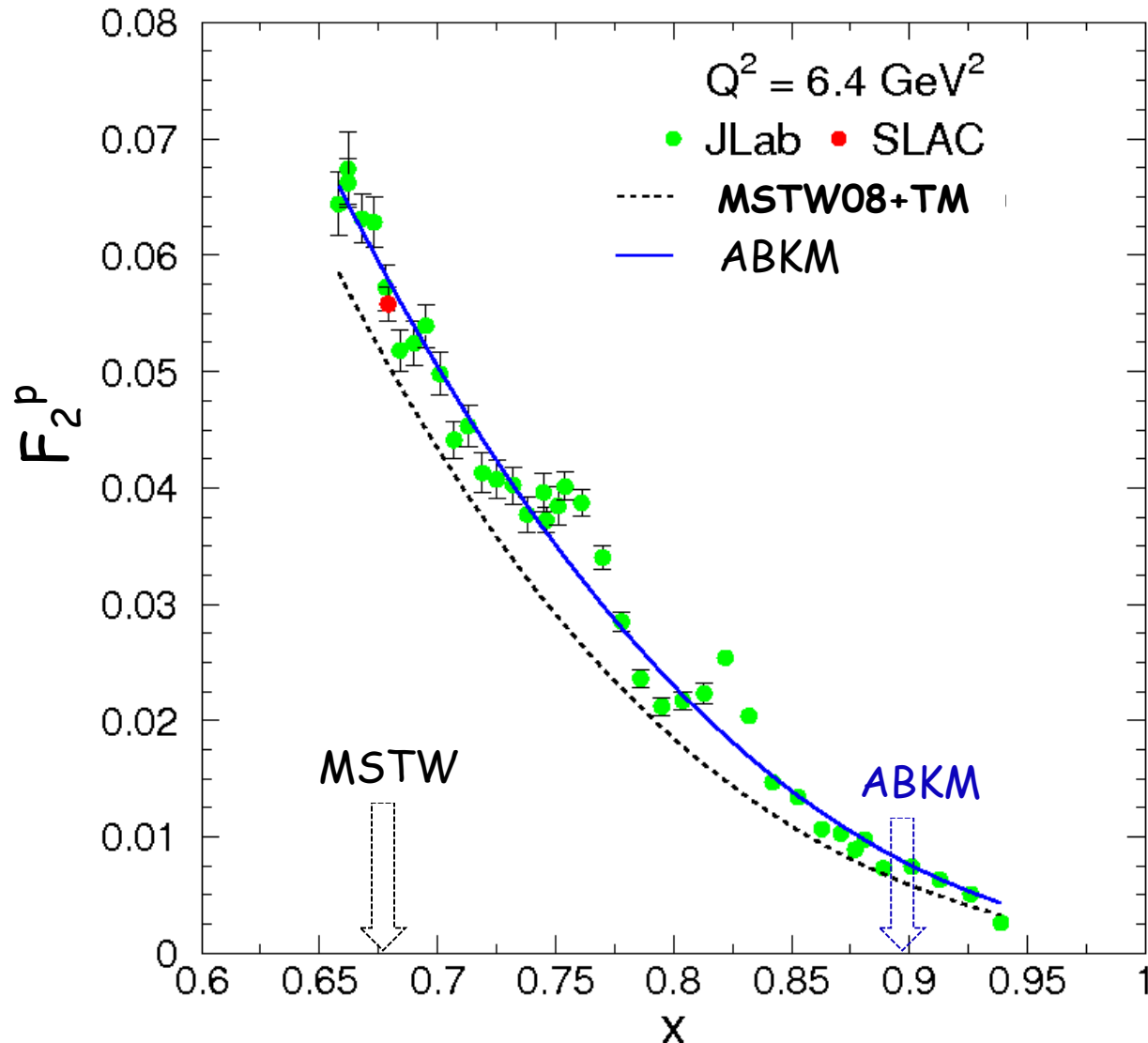
Accardi, Christy, Keppel, Melnitchouk, Monaghan, Morfín, Owens, Phys. Rev. D 81, 034016 (2010)

Accardi *et al.*, in preparation 

# Tests of Quark-Hadron Duality at large $x$

Resonance region data average to the QCD (beyond LT) calculation

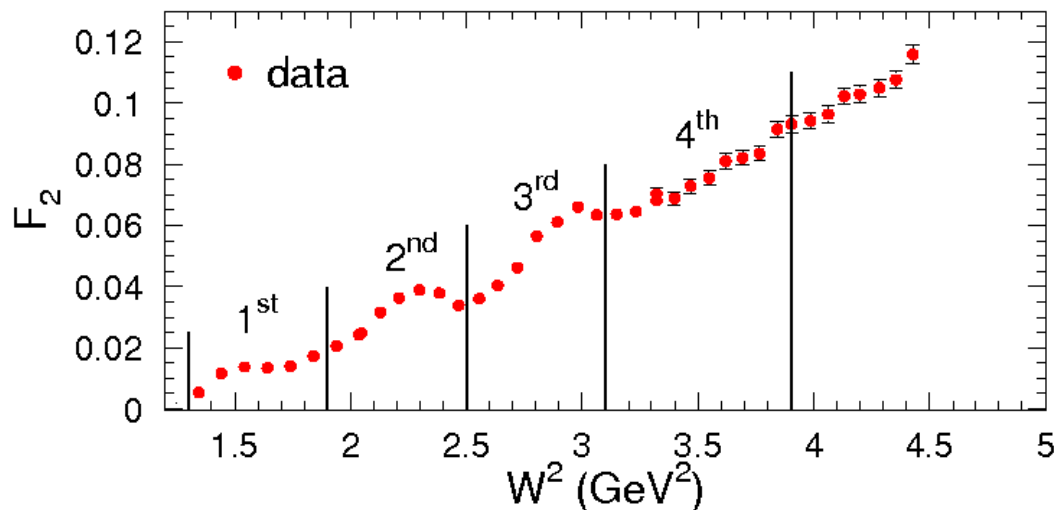
S. Alekhin, J. Blumlein, S. Klein, S. Moch, Phys. Rev. D 81, 014032 (2010)





# Quantitative Tests of Local Duality

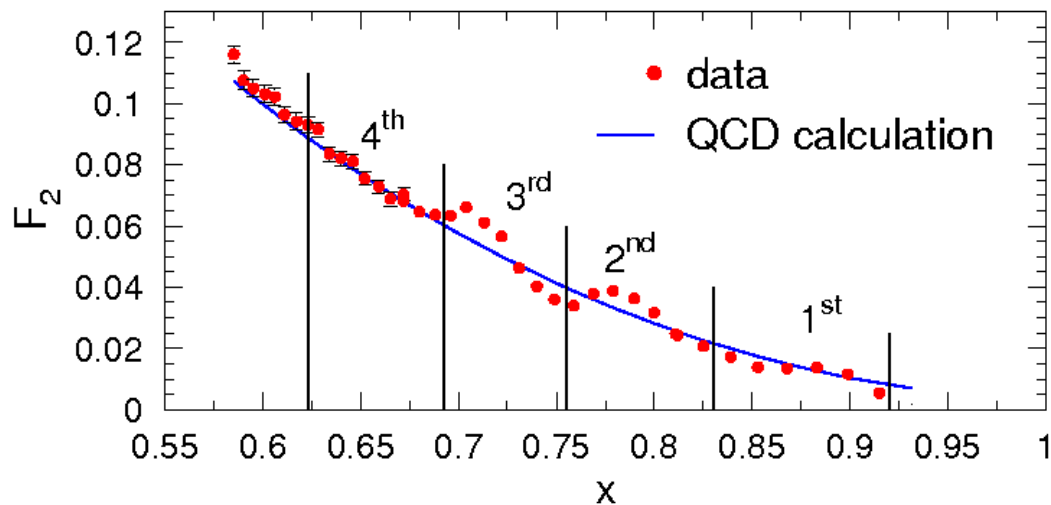
## 1) Delimit $W$ regions for duality tests



Region	$W^2_{\min}$	$W^2_{\max}$
1 <sup>st</sup>	1.3	1.9
2 <sup>nd</sup>	1.9	2.5
3 <sup>rd</sup>	2.5	3.1
4 <sup>th</sup>	3.1	3.9
DIS	3.9	4.5

$$x = Q^2 / (W^2 + Q^2 - M^2)$$

## 2) $F_2$ from data and QCD calculation



## 3) Calculate:

$$\frac{\int_{x_m}^{x_M} F_2^{data}(x, Q^2) dx}{\int_{x_m}^{x_M} F_2^{QCD calc.}(x, Q^2) dx}$$

# Quark-Hadron Duality in Proton $F_2^p$

$$\int_{x_m}^{x_M} F_2^{p,data}(x, Q^2) dx \bigg/ \int_{x_m}^{x_M} F_2^{p,param}(x, Q^2) dx$$

QCD calculation of Alekhin

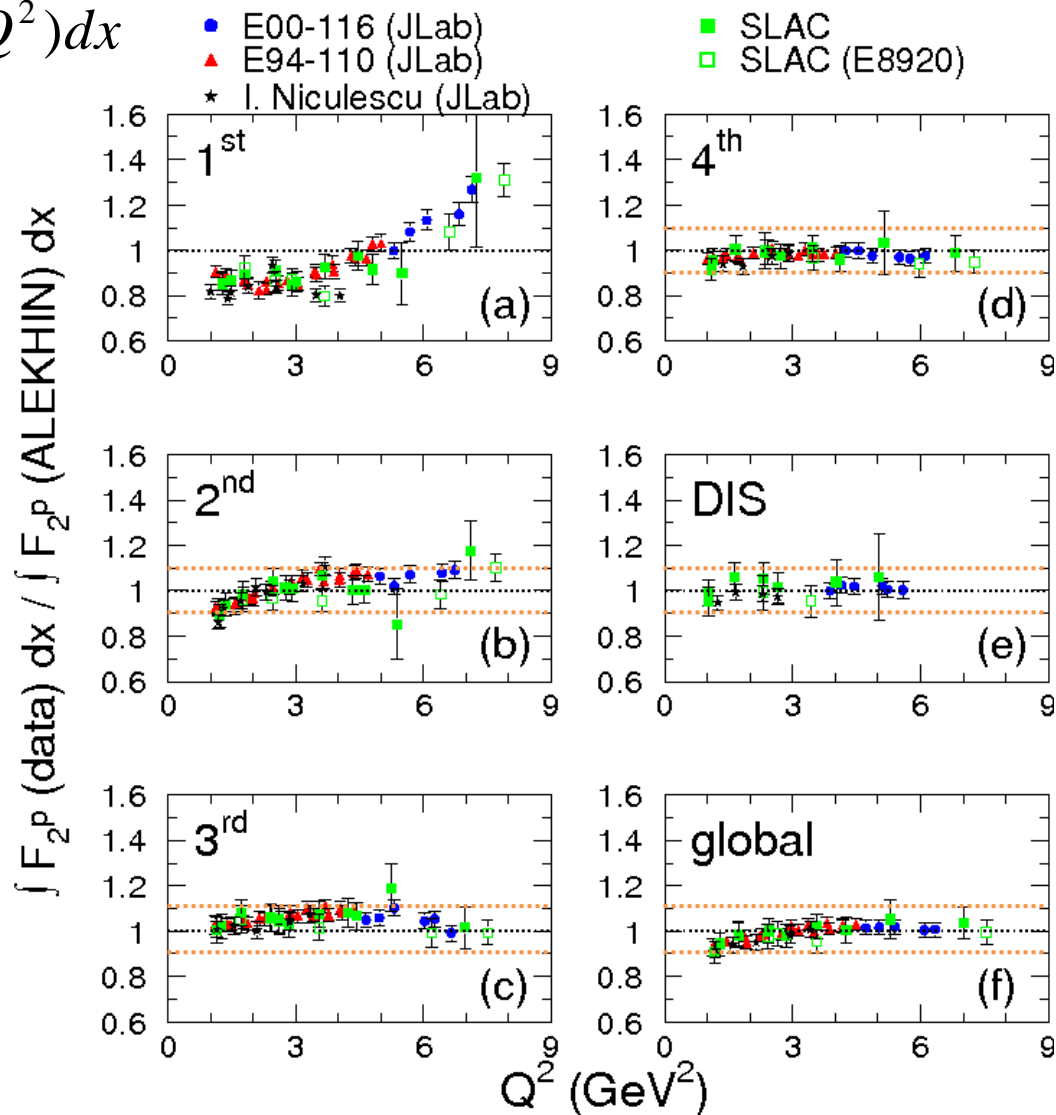
S. I. Alekhin, JETP Lett. 82, 628 (2005).

S. I. Alekhin, Phys. Rev. D 63, 094022 (2001)

✿ Within 10% : globally, 4<sup>th</sup>, 3<sup>rd</sup>, 2<sup>nd</sup>

✿ 1<sup>st</sup> : special case

- some models predict stronger violations of duality
- calculation based on handbag diagram may break at such low  $W$
- sits at the largest  $x$  (QCD fits poorly constrained) => difficult to test duality



S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)

# Quark-Hadron Duality in Neutron $F_2^n$

✿ Verify quark-hadron duality in  $F_2^n$

Need  $F_2^n$  in the resonance region...

Could use proton  $F_2^p$  and deuteron  $F_2^d$  **and**

**New method** to extract  $F_2^n$  from  $F_2^p$  and  $F_2^d$  : iterative procedure of solving integral convolution equations

Impulse Approximation:

$$F_2^A(x, Q^2) = \sum_{N=p,n}^{M_A/M} \int dy f_0^{N/A}(y, \gamma) F_2^N\left(\frac{x}{y}, Q^2\right) \quad \gamma = \sqrt{1 + \frac{4M^2 x^2}{Q^2}}$$

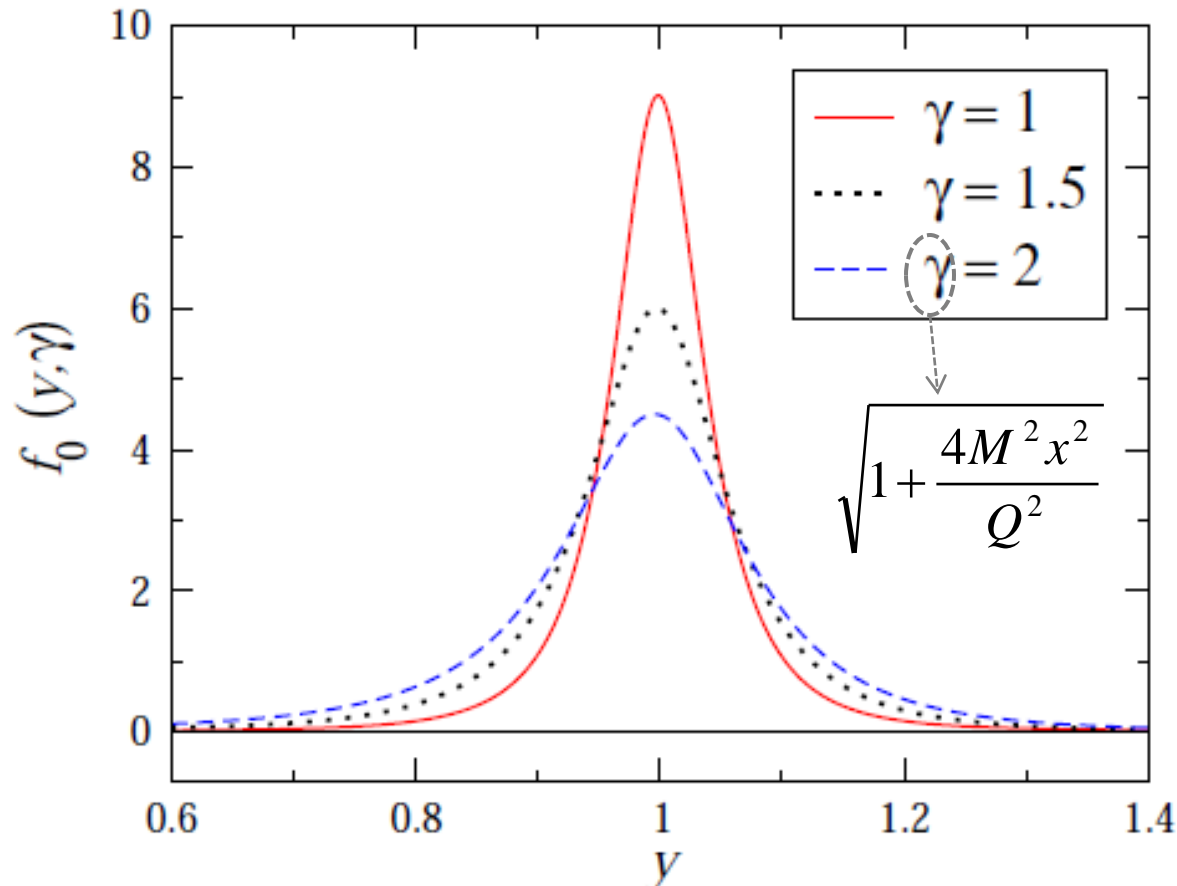
nuclear  $F_2$ 
light-cone momentum distribution of nucleons in nucleus (smearing function)
nucleon  $F_2$

# Smearing Function for $F_2^d$

- Smearing function evaluated in the weak binding approximation, **including finite- $Q^2$  corrections**

S.A. Kulagin and R. Petti, Nucl. Phys. A 765, 126 (2006)

Y. Kahn, W. Melnitchouk, S.A. Kulagin, Phys. Rev. C 79, 035205 (2009)



# Extraction Method

✿ We need  $F_2^n$  from:

$$\tilde{F}_2^n = F_2^d - F_2^{d(QE)} - \delta^{(off)} F_2^d - \tilde{F}_2^p$$

$$\tilde{F}_2^{n,p} = \int_x^{M_d/M} dy f(y, \gamma) F_2^{n,p} \left( \frac{x}{y} \right)$$

Additive extraction method: solve equation iteratively

$$f(y, \gamma) = \underbrace{N}_{\text{normalization of smearing function}} \delta(y-1) + \underbrace{\delta f(y, \gamma)}_{\text{finite width of smearing function}}$$

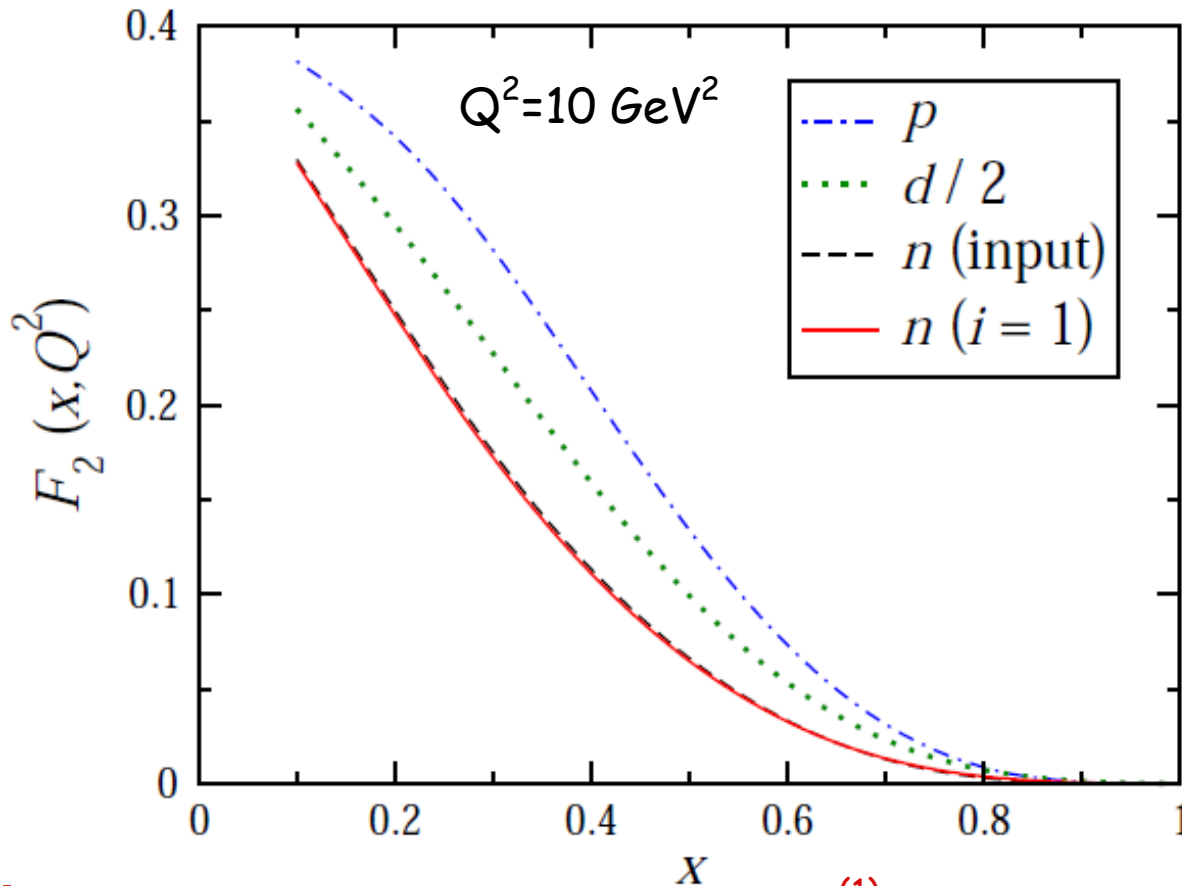
$$\tilde{F}_2^n(x) = NF_2^n(x) + \underbrace{\int_x^{M_d/M} dy \delta f(y, \gamma) F_2^n \left( \frac{x}{y} \right)}_{\text{perturbation}}$$

$$F_2^{n(1)}(x) = \underbrace{F_2^{n(0)}}_{\text{initial guess}}(x) + \frac{1}{N} \left[ \tilde{F}_2^n(x) - \int_x^{M_d/M} dy f(y, \gamma) \underbrace{F_2^{n(0)}}_{\text{initial guess}} \left( \frac{x}{y} \right) \right]$$

initial guess

# Application of Method to Smooth Curves

- Monotonic curves:  $F_2^p$  and  $F_2^n$  input from MRST;  $F_2^d$  is simulated using the finite- $Q^2$  smearing function
  - Additive method applied with initial guess  $F_2^{n(0)} = 0$



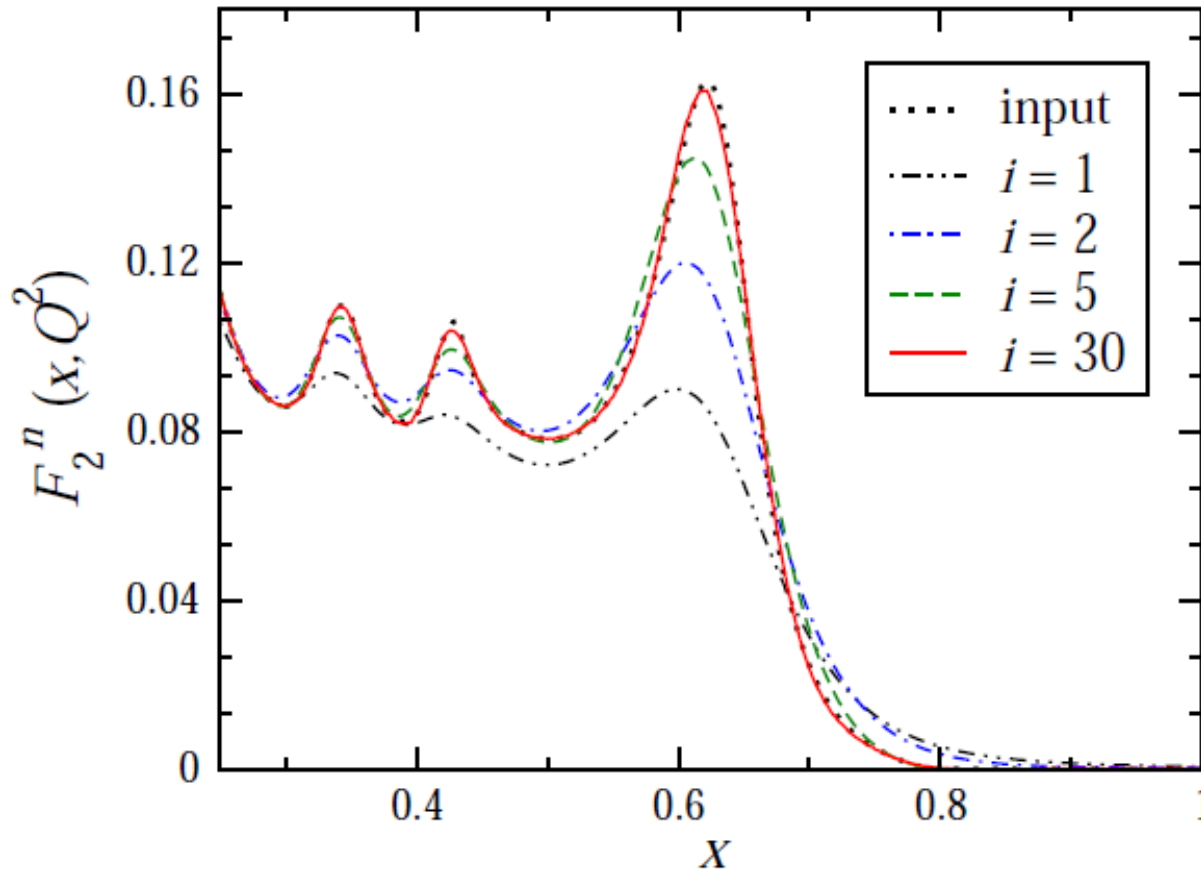
Y. Kahn, W. Melnitchouk,  
S.A. Kulagin, Phys. Rev. C 79,  
035205 (2009)

- Fast convergence: extracted  $F_2^{n(1)}$  almost indistinguishable from  $F_2^n$  input after only 1 iteration (smearing function sharply peaked around  $y = 1$ )

# Application of Method to Smooth Curves

✿ Curves with resonant structures:  $F_2^n$  input from MAID

- Additive method applied with initial guess  $F_2^{n(0)} = 0$

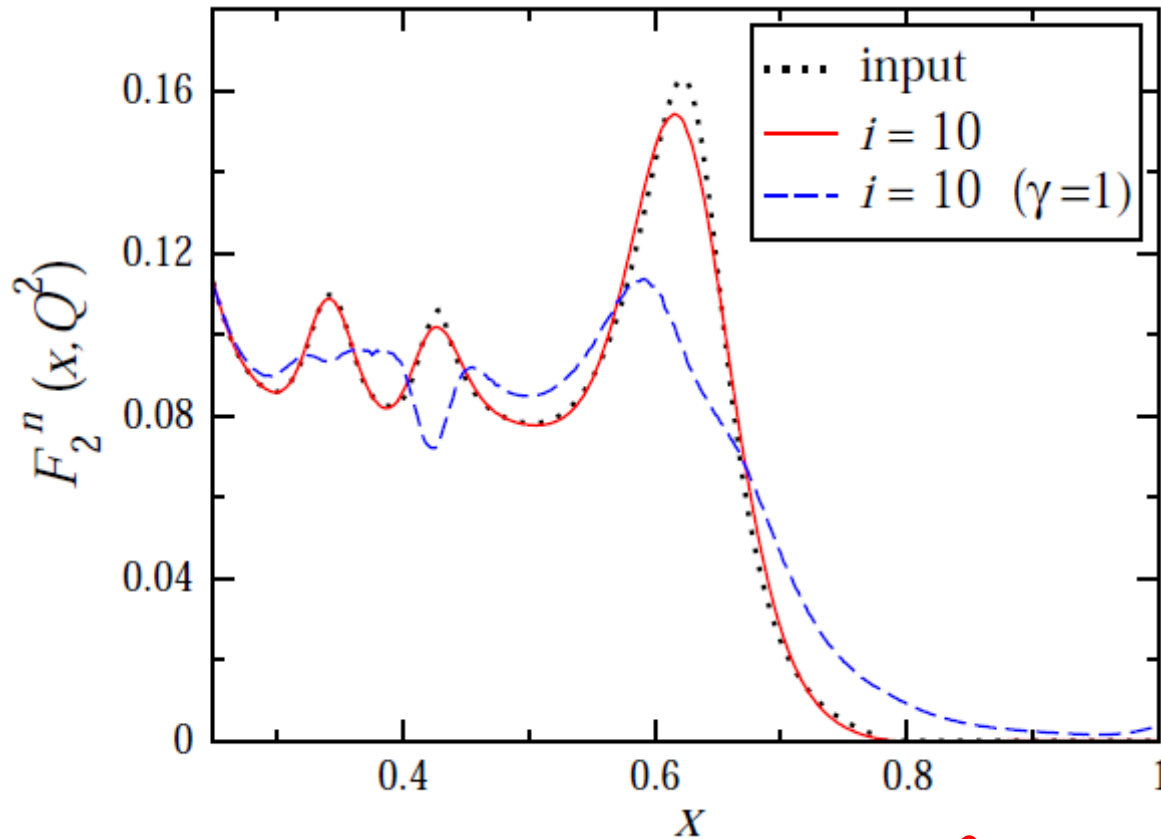


Y. Kahn, W. Melnitchouk,  
S.A. Kulagin, Phys. Rev. C 79,  
035205 (2009)

✿ After 1 or 2 iterations: resonant peaks clearly visible; after 5 iterations extracted result very close to “true” result

# Application of Method to Smooth Curves

- Essential to take into account  $Q^2$  effects in the smearing function
  - Additive method ( $F_2^{n(0)} = 0$ ):  $Q^2$ -dependent smearing function and  $Q^2$ -independent smearing function



Y. Kahn, W. Melnitchouk,  
S.A. Kulagin, Phys. Rev. C 79,  
035205 (2009)

- After 10 iterations: extraction with  $Q^2$ -dependent smearing function converges to the input; extraction with  $Q^2$ -independent smearing function does not



# Application of Method to Data

- Use proton and deuteron data at fixed  $Q^2$  (matched kinematics)

$$\tilde{F}_2^n(x) = F_2^d(x) - F_2^{d(QE)} - \delta^{(off-shell)} F_2^d(x) - \tilde{F}_2^p(x) \xrightarrow{\text{data}}$$

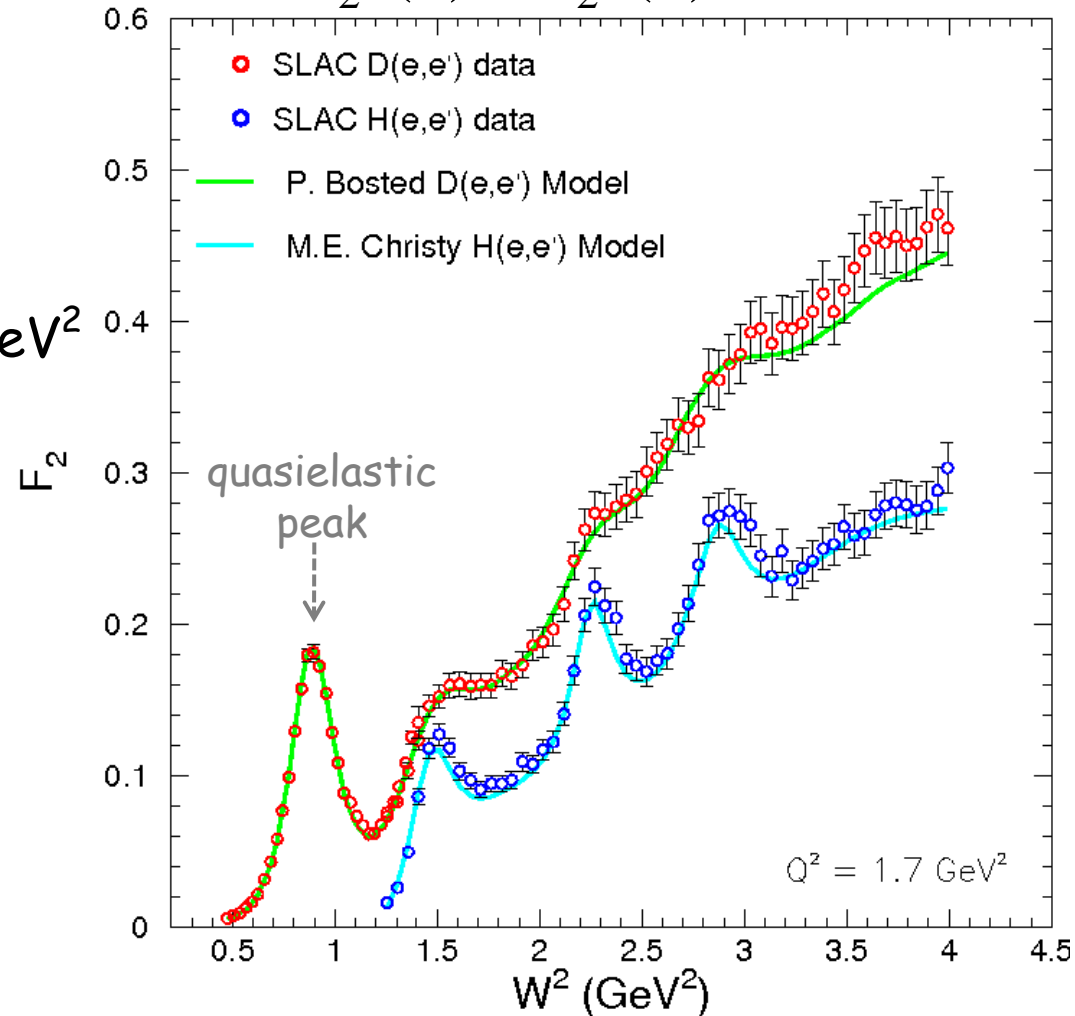
data

## Data:

SLAC at  $Q^2 = 0.6, 0.9, 1.7, 2.4 \text{ GeV}^2$

JLab (Hall C E00-116) at  $Q^2 = 4.5, 5, 5.5, 6.2, 6.4 \text{ GeV}^2$

- data at fixed  $Q^2 \Rightarrow$  bin-centering at cross section level using 2 different models



# Application of Method to Data

$$\tilde{F}_2^n(x) = \underset{\substack{\downarrow \\ \text{data}}}{F_2^d(x)} - \underset{\substack{\downarrow \\ \text{model}}}{F_2^{d(QE)}} - \overset{\text{model}}{\delta^{(off-shell)}} F_2^d(x) - \tilde{F}_2^p(x) \overset{\text{data}}{\dashrightarrow}$$

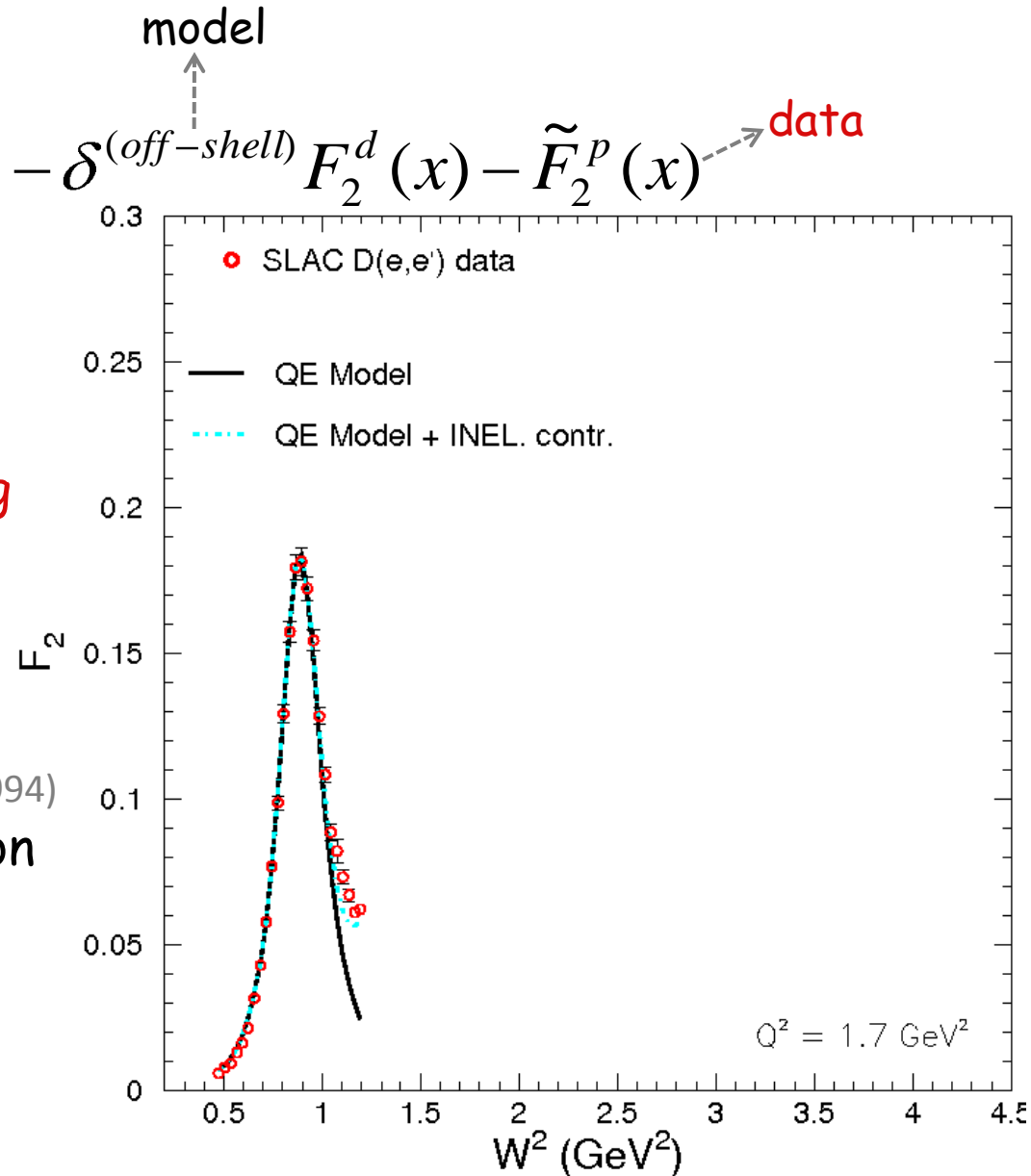
**QE:** extracted from data using **model** (form factors + **smearing function**)

**Off-shell corrections:**

- upper limit from model  $\sim 1.5\%$

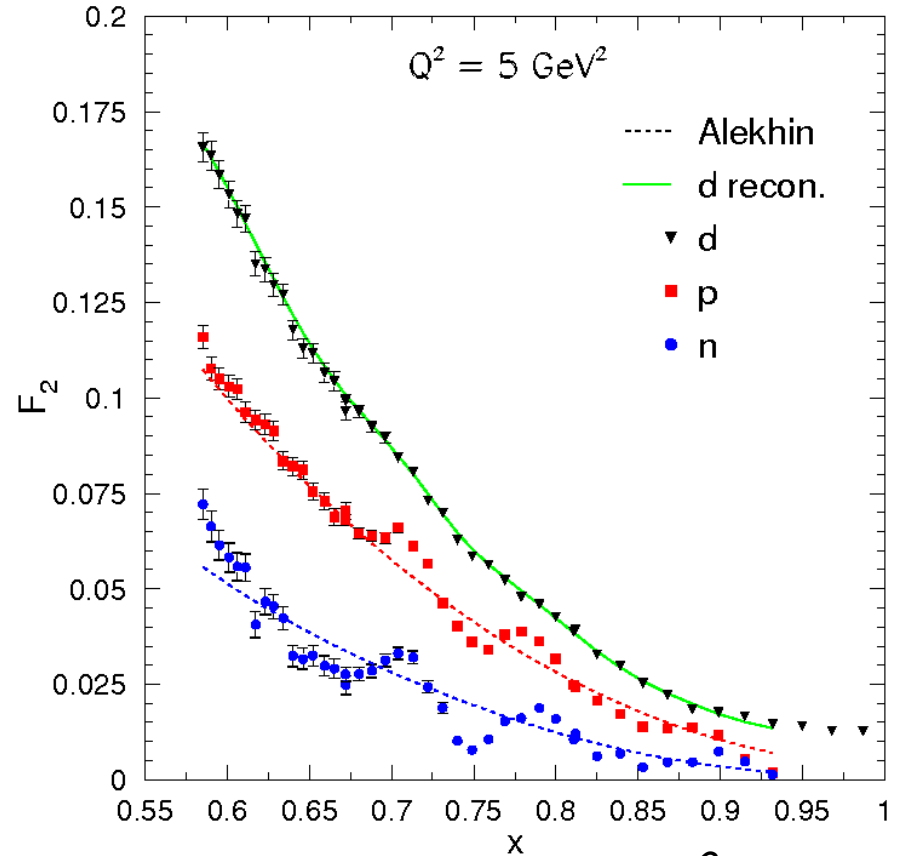
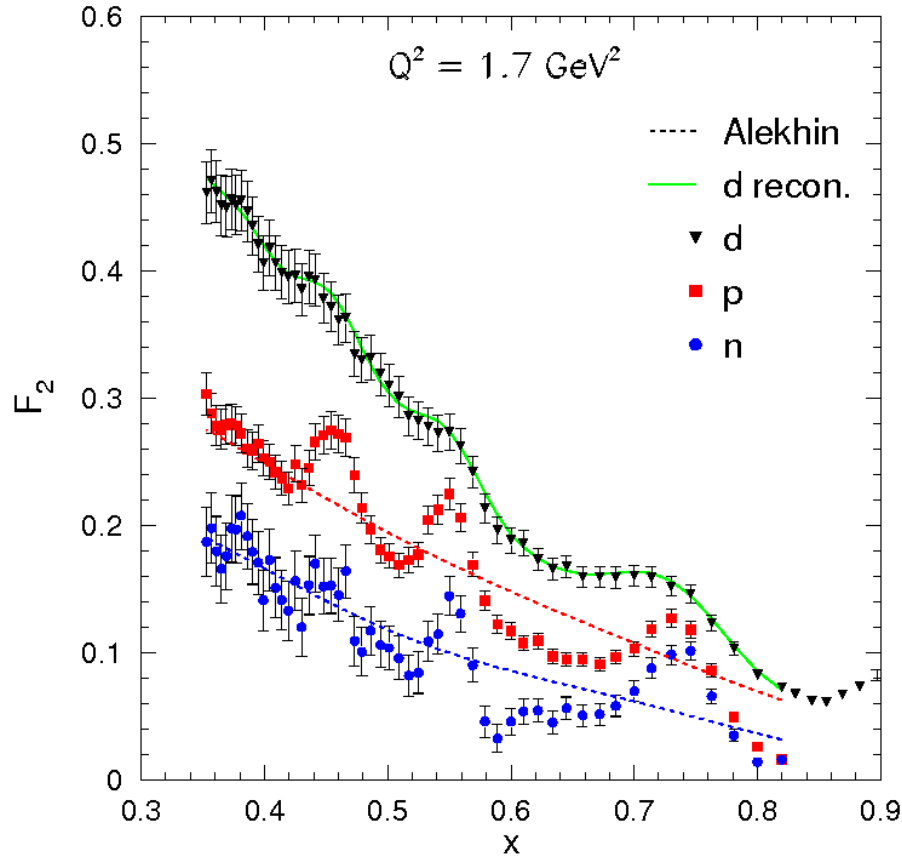
W. Melnitchouk *et al.*, Phys. Lett. B, 11 (1994)

- subtract  $\frac{1}{2}$  of model prediction
- assign 100% uncertainty to correction
- contributes  $< 2\%$  to total uncertainty on  $F_2^n$



# Application of Method to Data

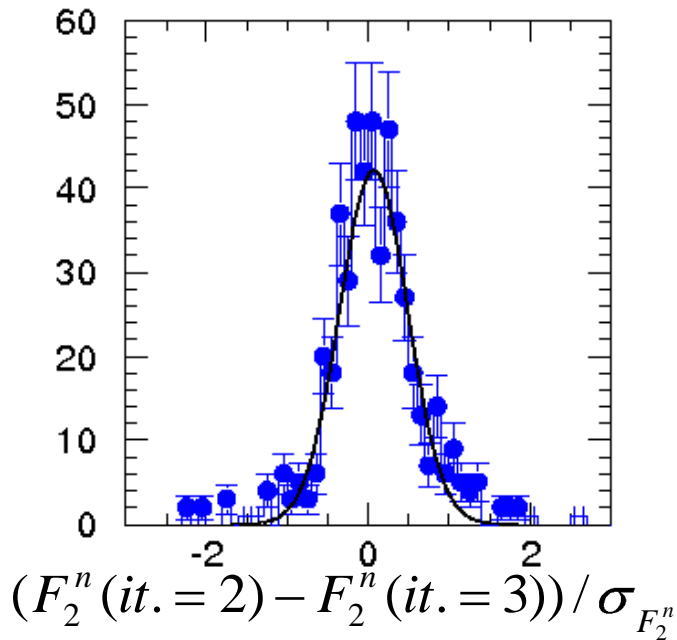
✿  $F_2^n$  extraction: initial guess  $F_2^{n(0)} = F_2^p$ ; number of iterations = 2



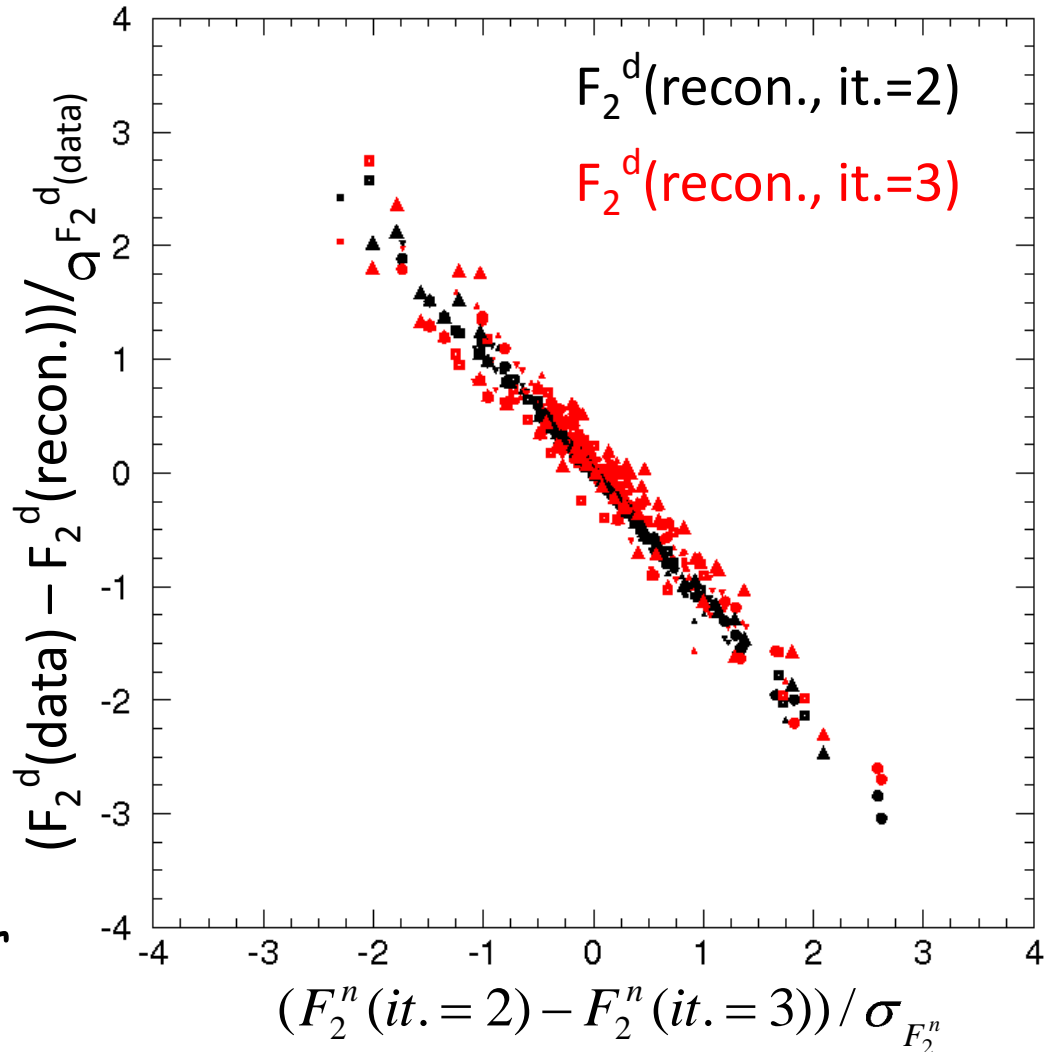
- $F_2^n$  in resonance region: 3 resonant enhancements (fall with  $Q^2$  at  $\sim$  rate as for  $F_2^p$ )
- $F_2^d$  reconstructed from  $F_2^p$ (data) and  $F_2^n$ (extraction)  $\sim F_2^d$ (data) after 2 iterations

# Application of Method to Data

- Study dependence of result on number of iterations: compare extractions with 2 and 3 iterations



- Small change in  $F_2^n$  between iteration 2 and 3
- Extracted  $F_2^n$  changes to bring  $F_2^d$  reconstructed closer to  $F_2^d$  data; small differences between iteration 2 and 3



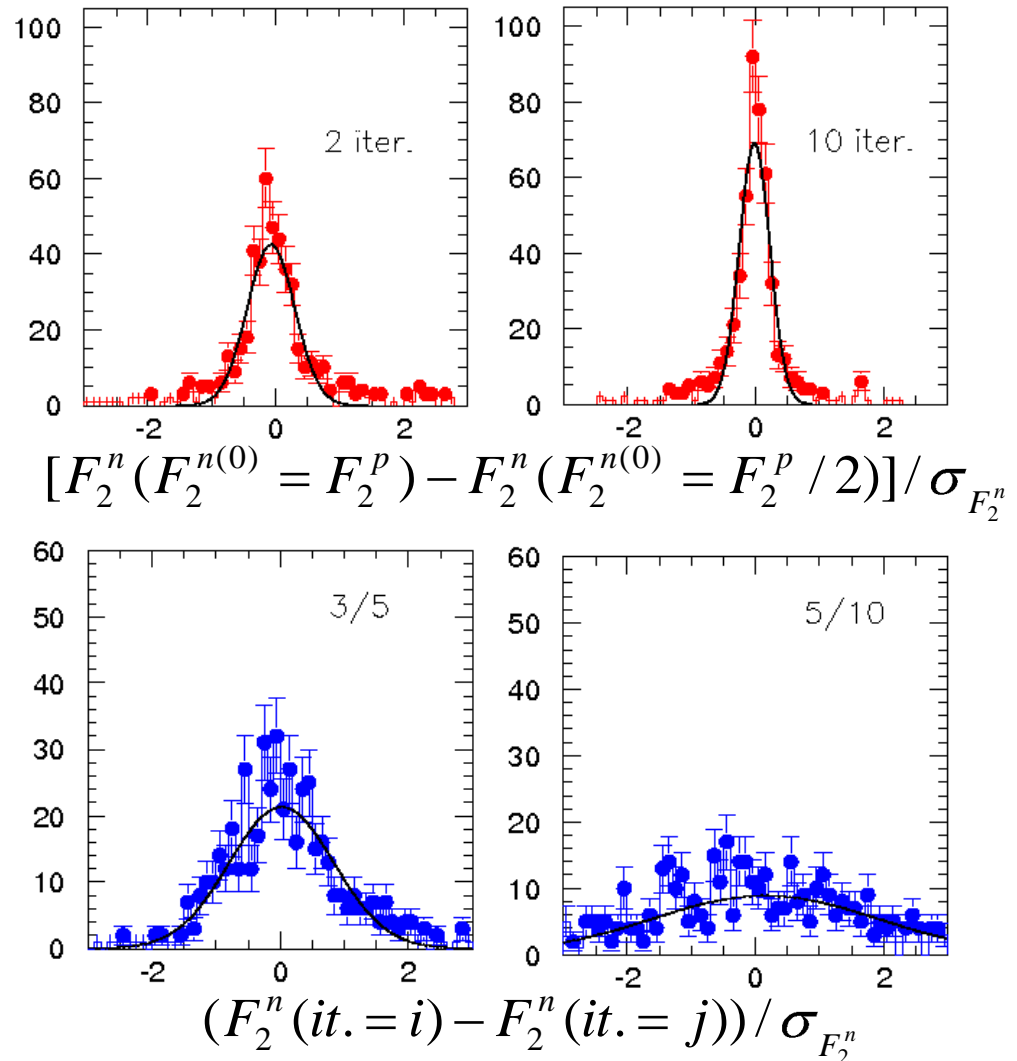
# Application of Method to Data

Study dependence of result on initial guess  $F_2^{n(0)}$  : compare  $F_2^n$  extracted with 2 different inputs for initial guess:  $F_2^{n(0)} = F_2^p$  vs  $F_2^{n(0)} = F_2^p / 2$

- After 2 iterations: only 6% of all data lay outside a  $2\sigma$  range

- Exercise caution with number of iterations: irregularities in data result in increased scattered in  $F_2^n$  with increasing number of iterations

S.P. Malace, Y. Kahn, W. Melnitchouk, in preparation



# Comparison to BoNuS Data

- Plots by Nathan Baillie

BoNuS:  $F_2^n$  data

Slava Tkachenko, Ph.D. thesis (2009)

Nathan Baillie, Ph.D. thesis (2009)

MALACE:  $F_2^n$  extracted from  $F_2^p$  and  $F_2^d$

S.P. Malace, Y. Kahn, W. Melnitchouk, C. Keppel, Phys. Rev. Lett. 104, 102001 (2010)

BOSTED

P.E. Bosted and M.E. Christy, Phys. Rev. C 77, 065206 (2008)

Thanks to Nate, Sebastian ☺  
and the BoNuS Collaboration

do not  
quote

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# Quark-Hadron Duality in the Neutron $F_2^n$

## Comparison: data to **ABKM**

S. Alekhin, J. Blumlein, S. Klein, S. Moch,  
Phys. Rev. D 81, 014032 (2010)

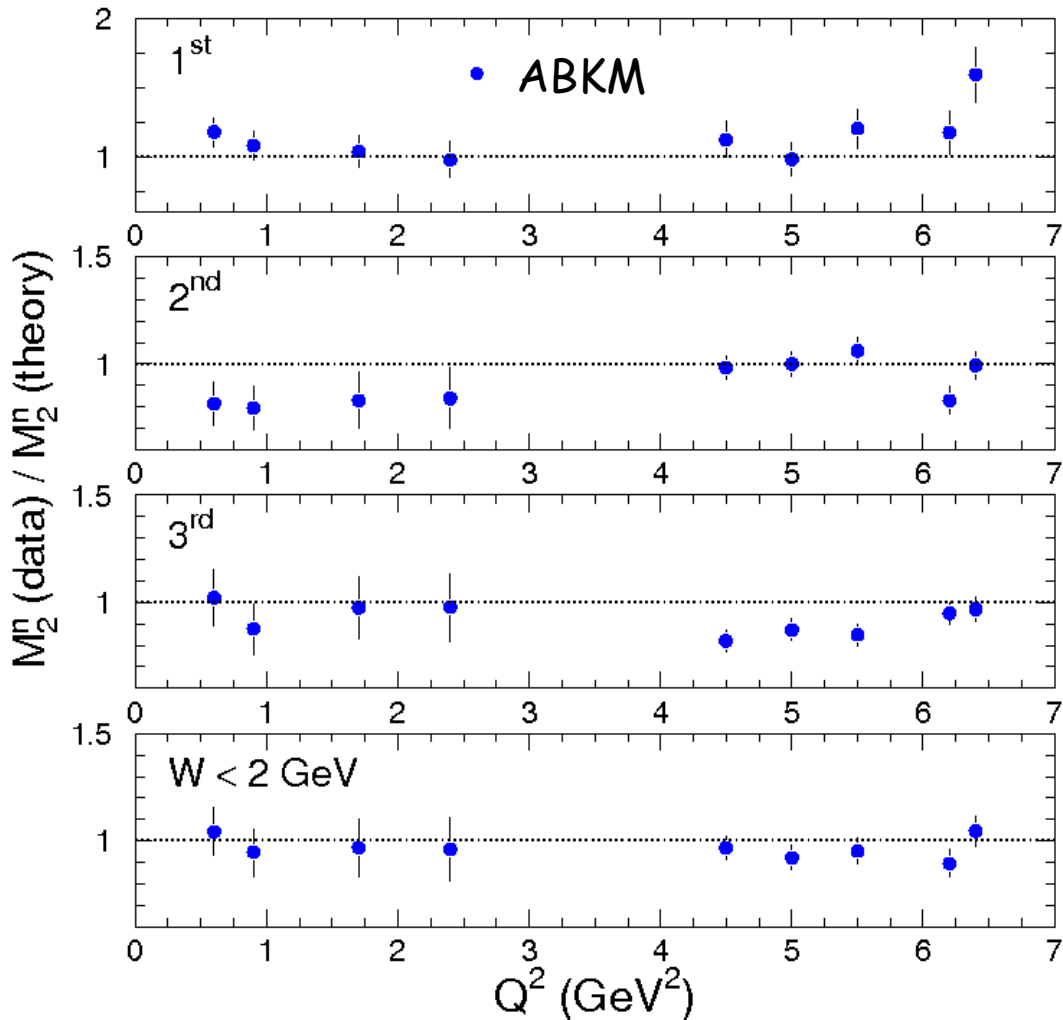
- 2<sup>nd</sup> and 3<sup>rd</sup> RES regions: agreement within **15-20%**, on average

- 1<sup>st</sup> RES region: agreement worsens at the highest  $Q^2$  (corresponds to the largest  $x$ )

- globally** remarkable agreement: within **10%**

S.P. Malace, Y. Kahn, W. Melnitchouk,  
Phys. Rev. Lett. 104, 102001 (2010)

$$\int_{x_m}^{x_M} F_2^{n,data}(x, Q^2) dx \bigg/ \int_{x_m}^{x_M} F_2^{n,param}(x, Q^2) dx$$

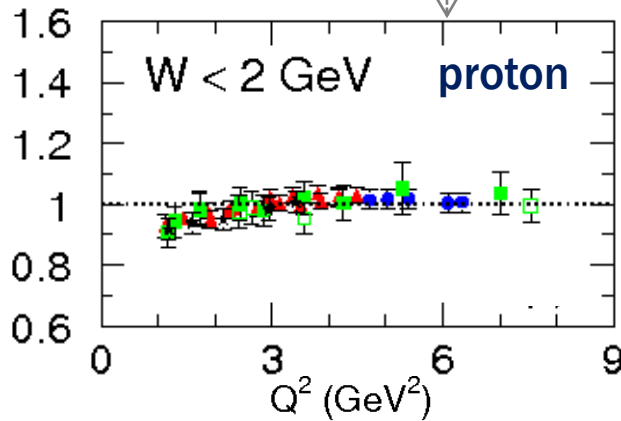


# Quark-Hadron Duality: Application

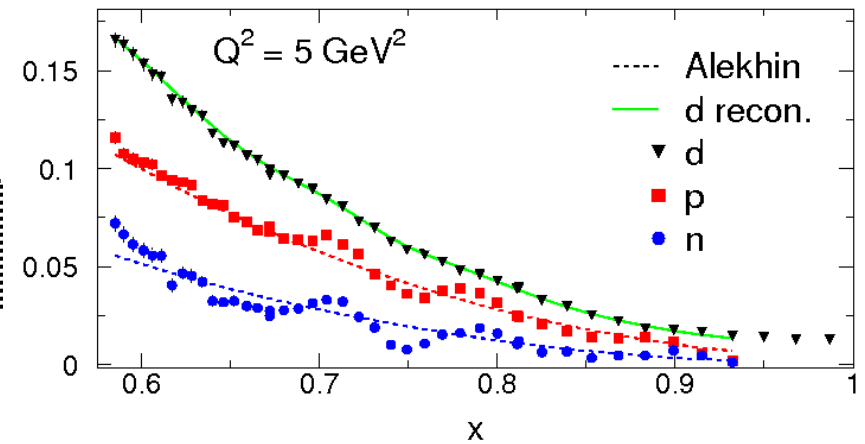
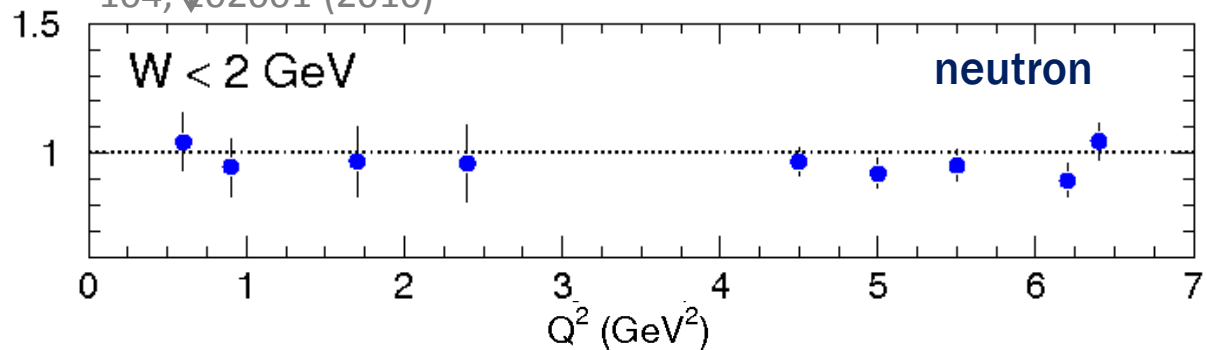
- Confirmation of **duality in both proton and neutron** => phenomenon not accidental but a general property of nucleon structure functions

$$\int F_2(\text{data})dx / \int F_2(QCD = \text{Alekhin})dx$$

S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)



S.P. Malace, Y. Kahn, W. Melnitchouk, C. Keppel, Phys. Rev. Lett. 104, 102001 (2010)



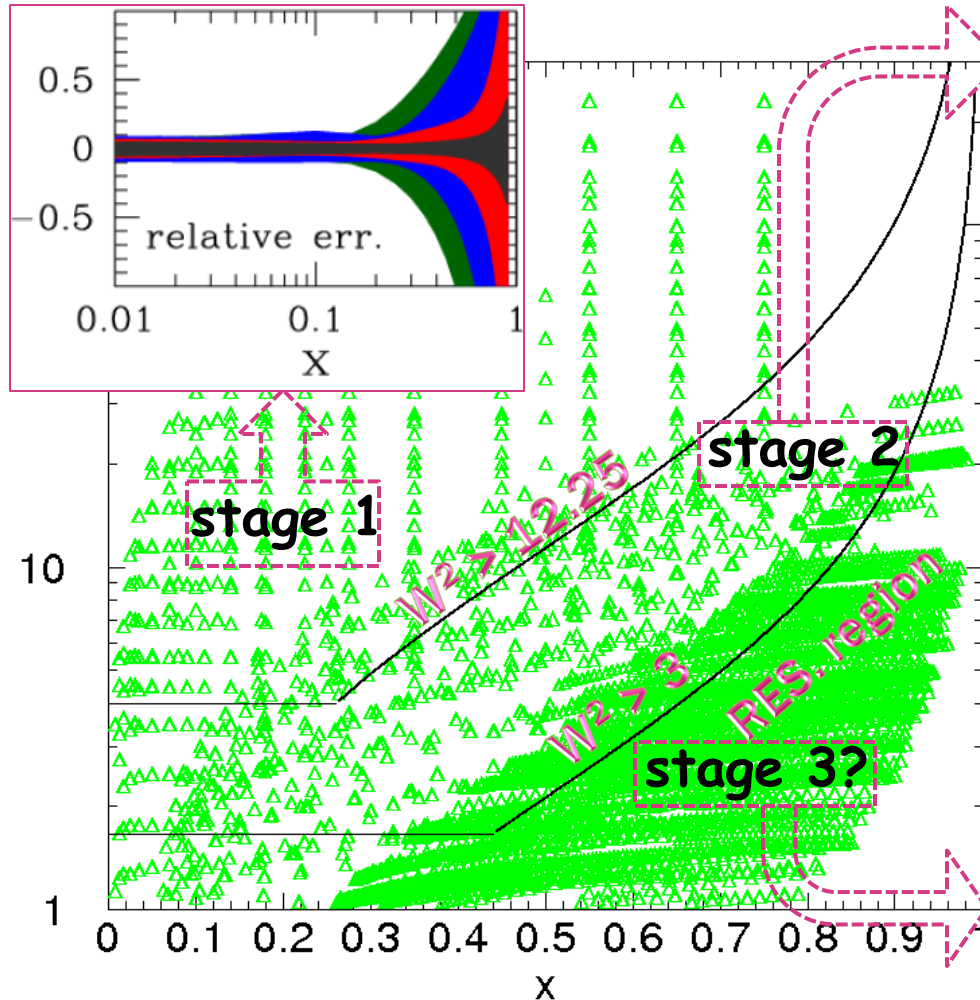
- Use averaged resonance region data ( $W^2 > 1.9 \text{ GeV}^2$ ) to extend PDFs extraction to the largest x

use... use... use... use...



# Quark-Hadron Duality: Application

➤ **Stage 1** (last few decades): LT calculations  $\Leftrightarrow$  PDFs constrained up to  $x \sim 0.7$  (CTEQ, MRST(MSTW), GRV, etc.)



➤ **Stage 2** (last decade): calculations beyond LT  $\Leftrightarrow$  PDFs constrained up to  $x \sim 0.8-0.9$

▪ **Alekhin et al.**

S. Alekhin, Phys. Rev. D 63, 094022 (2001)

.....  
S. Alekhin, J. Blumlein, S. Klein, S. Moch, Phys. Rev. D 81, 014032 (2010)

▪ **CTEQ6X**

Accardi, Christy, Keppel, Melnitchouk, Monaghan, Morfín, Owens, Phys. Rev. D 81, 034016 (2010)

Accardi et al., in preparation

➤ **Stage 3: future**

# Plans for Future: Quark-Hadron Duality

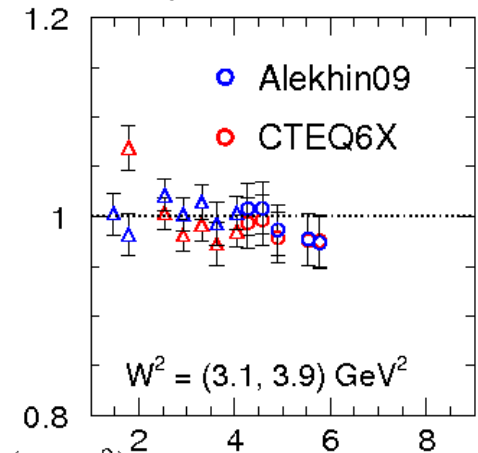
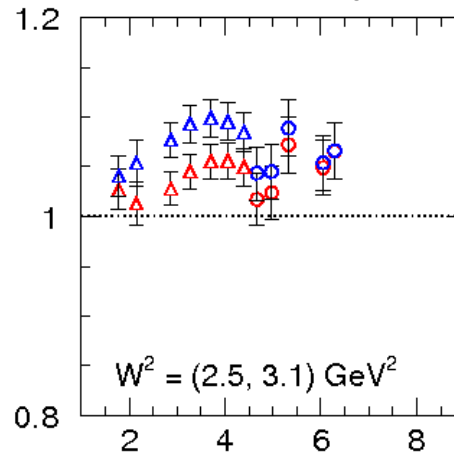
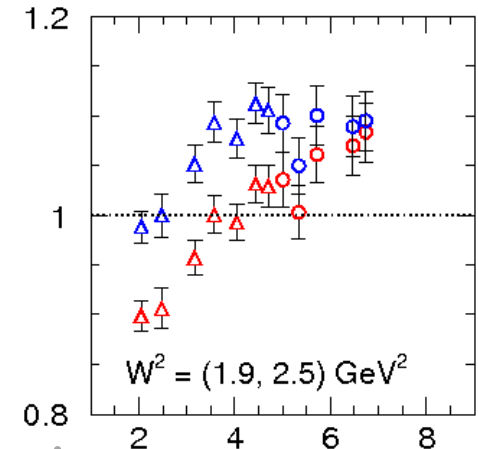
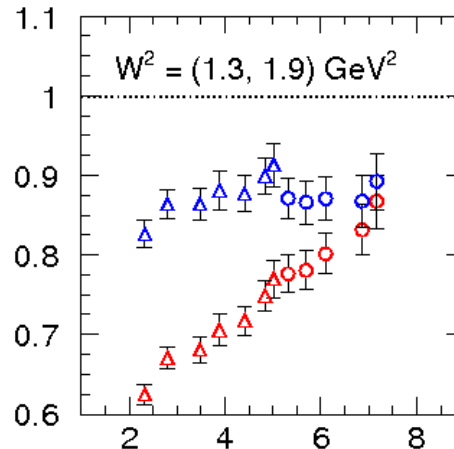
A. Accardi, S.P. Malace, in preparation

$$\int_{x_m}^{x_M} F_2^{p,data}(x, Q^2) dx \bigg/ \int_{x_m}^{x_M} F_2^{p,param}(x, Q^2) dx$$

Study sensitivity of quark-hadron duality ratios to various prescriptions for inclusion of:

- Higher Twist: additive vs multiplicative; HT(proton) same or different than HT(neutron)
- Target Mass Corrections: OPE, CF...
- etc.

M<sub>p</sub>(data)/M<sub>p</sub>(theory)



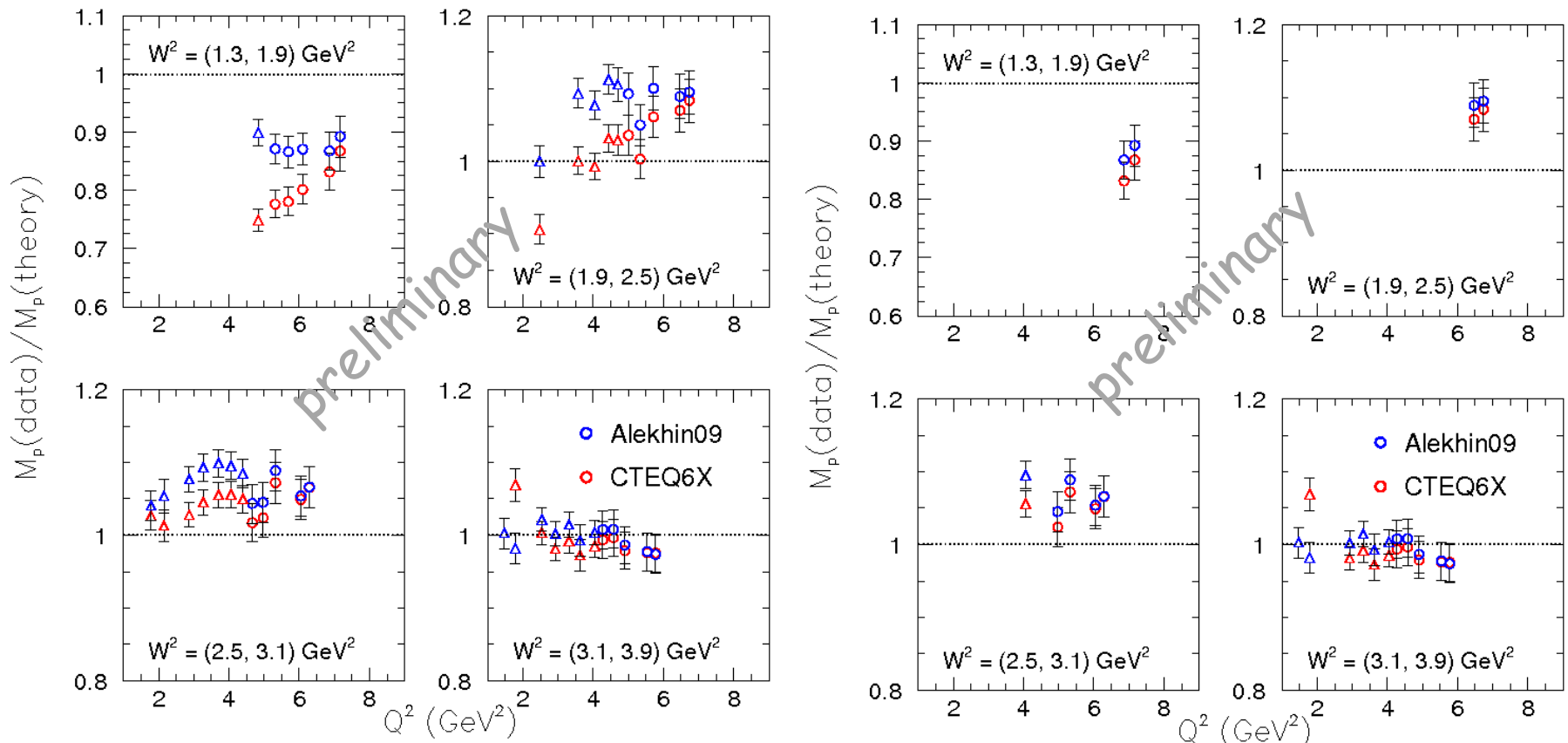
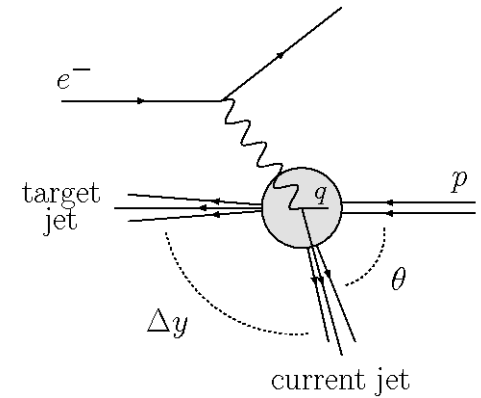
preliminary

$Q^2 (\text{GeV}^2)$

# Plans for Future: Quark-Hadron Duality

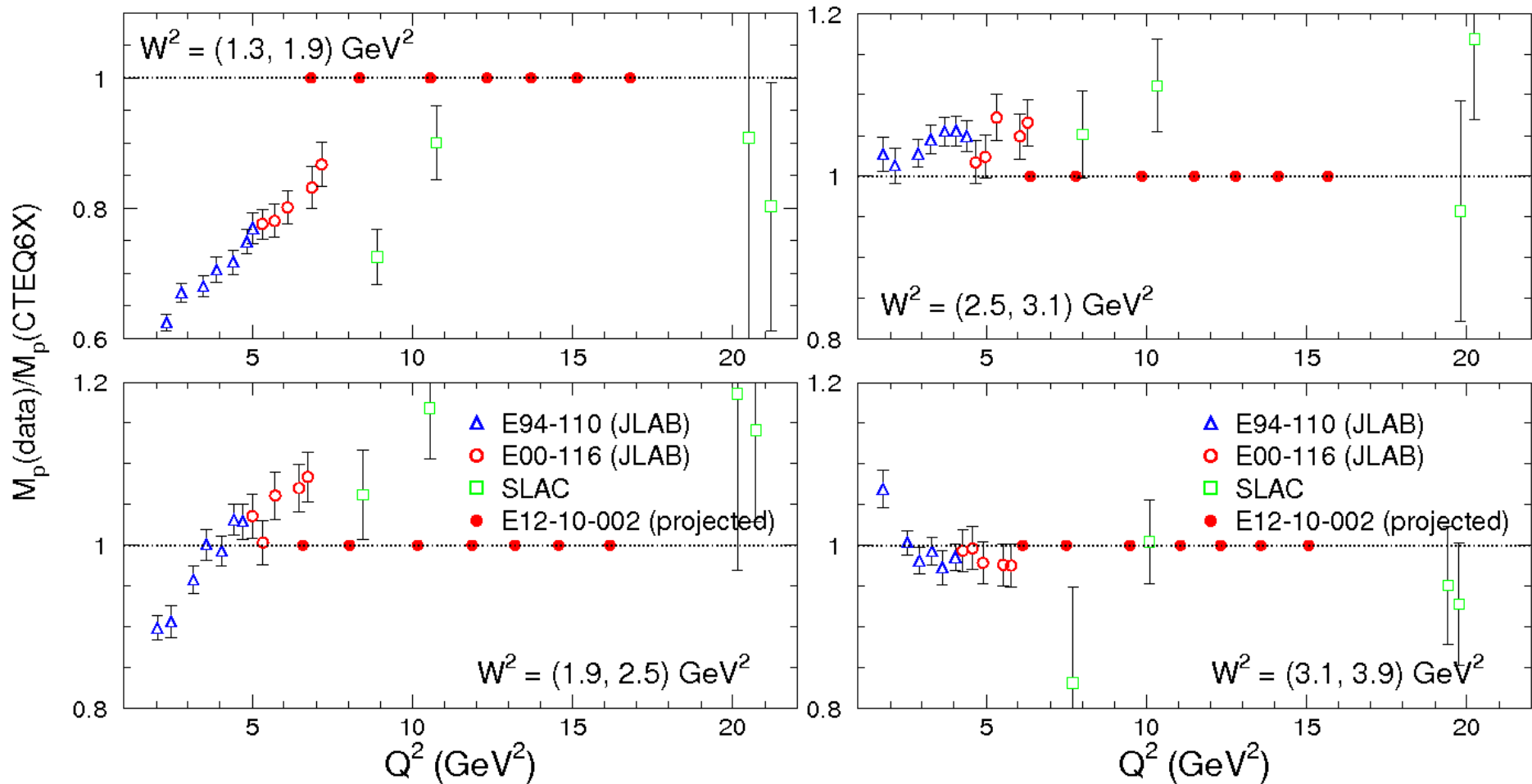
A. Accardi, S.P. Malace, in preparation

- Study applicability of QCD calculation at low values of  $W$ ; criterion: separation between target jet and current jet



# Duality @ 12 GeV: E12-10-002

- Extend proton and deuteron  $F_2$  structure function precision measurements to larger  $x$  and  $Q^2$  in the resonance region and beyond up to  $W^2 \sim 9 \text{ GeV}^2$ ,  $Q^2 \sim 17 \text{ GeV}^2$  and  $x \sim 0.99$





# Extra Slides

